COMPUTATIONAL STUDY OF HEAT TRANSFER IN THE LAMINAR FLOW OF GAS IN A FLAT CURVILINEAR CHANNEL WITH SUCTION AND INJECTION

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We have made a numerical study of the effect of normal suction and injection of gas through the walls of a flat curvilinear channel on the total pressure loss coefficient and local heat transfer (Re < 1000; M = 0.5).

Questions of heat transfer during the flow of a high-temperature gas through a curved channel or one with a sharply varying cross section are important in view of their great practical significance. Such flows have a complex gasdynamic structure; for example, they have closed and open separation regions, and secondary flows in a cross section of the channel. These features lead to additional total pressure losses and promote an increase of decrease of heat transfer to the walls in the corresponding regions. These effects depend on many factors: the Re, M, and Pr numbers, the deflection of the flow, the sizes of projections or indentations on the walls, the degree of turbulence of the flow, the temperature factor, the properties of the gas, etc.

There are many experimental [1-4] and computational [5-11] papers on various aspects of the heat-transfer problem in flows with circulation and separation regions. The development and investigation of methods of controlling heat transfer [7, 9, 11-13], for example by injection, suction, forced cooling of the walls, etc., are very important.

In the present article we employ the numerical method developed in [11] to study control of gasdynamic and heat-transfer characteristics of the flow in a sharply curved flat channel by gas injection and suction. We consider laminar flow (Re < 1000) of a perfect gas under subsonic conditions (M < 1). We study the effect of the rate of suction and its location on the total pressure loss and heat flows to the wall. We determine the most effective location of suction for the channel studied.

The data obtained are in qualitative agreement with the results of gasdynamic studies of a curvilinear channel [13] and thermal studies of flows with separations [6-9].

The characteristic feature of the actual flows studied — a high temperature of the gas (1000-2000°K) which requires taking account of the actual properties of the gas — is a strong dependence of the transfer coefficients on the enthalpy, and the special form of the equation of state. Without going into details, we indicate a divergence of up to 20% from the results for a perfect gas. The qualitative conclusions agree.

We consider the flow of a hot compressible gas in a flat curvilinear channel of constant height H. The curved section of the channel has a 90° bend, and its inner surface has a radius of curvature R. The straight inlet and outlet stabilizing sections are long enough to ensure developed flow at the channel outlet and flow with specified parameters at the inlet. This channel geometry was studied earlier, for example in [11], where the optimum lengths of the straight portions were determined in a numerical experiment.

The system of equations for describing the flow of a gas in such a channel includes the complete Navier—Stokes equations, the equation of continuity, the energy equation, and the equation of state [11]. A numerical method for solving this system of equations is described in [11]. It uses natural variables (density ρ of the gas, temperature T, static pressure p, and velocity $\mathbf{V}$ with a longitudinal component u and transverse component v), a hybrid scheme, and an iterative process of solution with underrelaxation. In the straight portions of the channel a Cartesian coordinate system is used with the x axis along one wall and the y axis normal to the walls. In the curved section polar coordinates r and θ are used with θ measured from the beginning of the bend (Fig. 1). The common longitudinal coordinate for all parts is the arc length s of the inner wall, measured from the channel inlet.

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The computational region contains 32 × 46 nodes formed in the nonuniform subdivision of the channel region by lines parallel and perpendicular to the walls. The net is compressed near the walls and in the vicinity of the bend.

Laminar (Re < 1000) flows of a hot gas in a curvilinear channel can be efficiently studied by the numerical method developed. These flows are characterized by closed circulation regions which are an additional source of total pressure loss ξ, and produce a redistribution of local heat fluxes q_w to the wall [11]. Let us recall their characteristic position in the channel under consideration. On the inner wall the region begins at the end of the curved section and ends downstream in the rectilinear part of the channel. On the outer wall the separated flow region is formed at the beginning of the bend, and lies completely within the curvilinear section (Fig. 1). The circulation region is substantially more intense on the inner wall than on the outer wall; its thickness may reach 25% of the height of the channel, and its length 2-3 diameters. The length and thickness of these regions depend on many parameters of the problem: the Reynolds number Re, the wall temperature T_w, the curvature Cu = H/R of the channel walls, etc.

In what follows we investigate how such a practically important factor as normal suction (or injection) of gas through the walls affects the flow characteristics. Both the intensity of the suction and its location relative to the circulation regions are important.

Suppose gas is injected into the flow with a normal velocity v_{nw} = const over a limited portion of a wall $s_1 \leq s \leq s_2$. Since the normal **n** is directed into the flow, $v_{nw} > 0$ corresponds to injection, and $v_{nw} < 0$ to suction. The flow rate of the injected gas $G_i = \int_{s_1}^{s_2} \rho_w v_{nw} ds$ is not a linear function of s, since $\rho_w \neq$ const, but is determined from the solution of the problem.

We formulate the complete boundary conditions for the whole problem: at the channel inlet (s = 0), at its outlet ($s = s_3$), and on the walls (y = 0, y = H). In the problem under consideration they have the following form:

$$
\begin{gathered}
s=0,\ 0<y<H: T=T_0=\text{const},\ \rho=\rho_0=\text{const},\\
v=0,\ u=u_0\{1-(2y/H-1)^2\};\\
s=s_3,\ 0<y<H: \partial T/\partial s=\partial v/\partial s=\partial u/\partial s=0;\\
y=0 \text{ and } y=H,\ 0\leqslant s\leqslant s_3: u=0,\ T=T_w,\ v=0;\\
s_1\leqslant s\leqslant s_2: v=v_w.
\end{gathered}
\tag{1}
$$

We assume the gas is perfect, i.e., we use the equation of state for a perfect gas with constant specific heats c_{p0} and c_{v0}, but with temperature-dependent viscosity $\mu = \mu_0 T/T_0$ and thermal conductivity $\lambda = \lambda_0 T/T_0$.

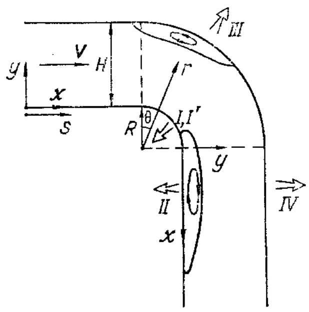
We used dimensionless variables in performing the calculations. We took the characteristic values of quantities in the middle of the cross section of the channel inlet. The following dimensionless quantities were assumed constant in the calculations:

$$
\begin{gathered}
\text{Re}=Hu_0\rho_0/\mu_0=500;\ \text{Pr}=c_{p0}\mu_0/\lambda_0=0.75;\ \gamma=c_{p0}/c_{v0}=1.4;\\
\overline{T}_w=T_w/T_0=0.8;\ \text{M}=u_0/\sqrt{\gamma(c_{p0}-c_{v0})T_0}=0.5.
\end{gathered}
\tag{2}
$$

The curvature of the inner wall Cu = H/R = 2 and its angle of bend $\theta = \pi/2$ were also constants. These parameters enter the matching conditions for the solutions at the junctions of the rectilinear and curvilinear portions of the channel.

The following variable dimensionless parameters relate to suction (injection):

$$
S=s_1/H,\ \eta=(s_2-s_1)/H,\ g=G_i/G_0\ \left(G_0=\int_0^H \rho u|_{s=0}dy\right).
\tag{3}
$$



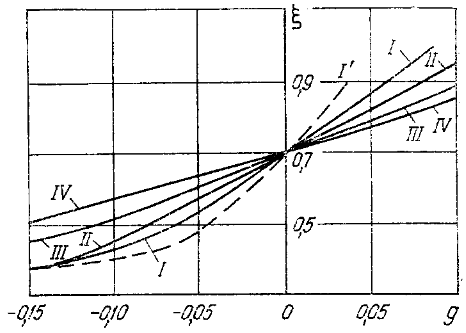


Fig. 1 Fig. 2

Fig. 1. Formulation of the problem.

Fig. 2. Total pressure loss coefficient ξ as a function of the relative flow rate of sucked (g < 0) and injected (g > 0) gas at various locations of suction and injection (for regimes I-IV and I' see Fig. 1).

They characterize the location, length, and intensity of suction (g < 0 for suction, g > 0 for injection). In specific calculations we used the independent parameter $V_w = v_w/u_0$ instead of the parameter g.

Studies of flow for g = 0 and variable Re, $\bar{T}_w$, and Cu were reported in [11].

We consider two practically important characteristics of the flow: the total pressure loss coefficient ξ and the local heat flux $\bar{q}_w$ to the walls. Calculations show that the latter is the most important thermal characteristic of the flow. For g ≠ 0 it may vary severalfold, whereas the integrated heat flux to the whole length of wall varies negligibly. For g ≠ 0, $\bar{q}_w$ is redistributed over the wall, forming regions with maximum and minimum values of q_w. It is important to find these regions.

In the present article the dimensionless characteristics mentioned are defined as follows:

$$\xi = 2\,(\rho_0 u_0^2 H)^{-1} \int\limits_0^H \{(p + \rho v^2/2)_0 - (p + \rho v^2/2)_1\}\, dy; \tag{4}$$

$$\bar{q}_w = (c_{p0} T_0 \rho_0 u_0)^{-1} \lambda_w \partial T/\partial n. \tag{5}$$

Expression (4) corresponds to an incompressible gas, and is sufficiently accurate for our case (M = 0.5). The subscripts 0 and 1 denote cross sections at inlet and outlet. Formula (5) is one way of writing the Stanton number.

Figure 2 shows ξ as a function of g for various S and η. Curves I and I' correspond to suction (injection) on the inner wall in the bend where the circulation region begins. For curve I the suction region occupies the whole wall in the bend; for curve I' the region is localized close to the beginning of the separation region, which is located at the end of the section, and occupies only 30% of the length of the wall of the bend. For curve II the suction region is on the same wall completely inside the separation region, i.e., immediately after the curved section, and has a length 0.6 of the channel diameter. Curves III and IV correspond to the outer wall — in the bend and beyond it. In Fig. 1 the double arrows indicate the approximate locations of the various suction regimes.

Figure 2 illustrates the familiar fact that losses decrease during suction (g < 0), and increase during injection (g > 0). The most effective decrease of ξ occurs for curves I and I', i.e., during suction at the beginning of the strongest separation. The divergence of curves I and I' at small |g| shows that the more important parts of the suction are located in the immediate vicinity of the separation point. For large |g| curves I and I' approach one another and merge when the circulation region disappears into the suction region. We note that for suction the circulation region decreases and finally disappears, while for injection it is thickened.

The effect of an appreciable decrease in ξ for small relative flow rates of the sucked gas corresponding to situations I and I' was studied experimentally in [13].

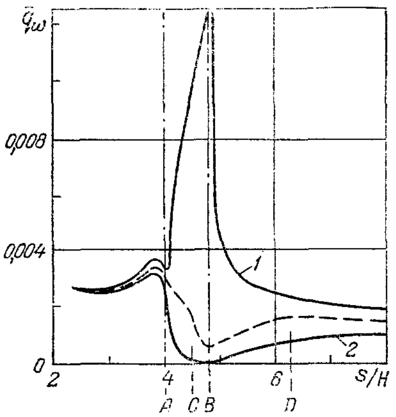


Fig. 3

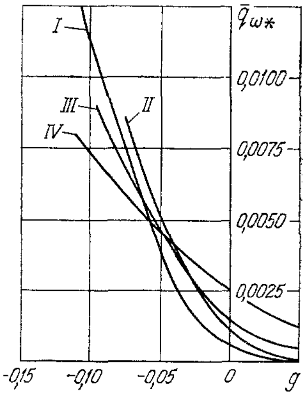


Fig. 4

Fig. 3. Distribution of dimensionless magnitude of the heat flux $\bar{q}_w$ along the length of the inner wall for g = 0 (open curve), g = −0.11 (curve 1), and g = 0.09 (curve 2). Suction and injection occur on this wall in the curved section.

Fig. 4. Dependence of $\bar{q}_{w*}$ (the characteristic value of the heat flux at a certain point of the wall) on g for suction and injection regimes I-IV.

It should be noted that while ξ is important as an experimentally measurable quantity, it is not an adequate characteristic property of a channel for $g \neq 0$. It includes the total pressure loss carried away from the channel by the sucked gas. In order to separate the characteristics of the property ξ' of the channel, proportional to the integral of the viscous dissipation, it is necessary to set up balance relations for the flow of total pressure through the whole closed boundary of the channel. Balance relations for a channel with suction were proposed in [13]. Such calculations were performed and showed that $\xi'(g)$ has qualitatively the same behavior as $\xi(g)$.

We turn to the heat-transfer characteristics. Figure 3 shows the distribution of $\bar{q}_w(s/H)$ on the inner wall for case I with $g \neq 0$ (solid curves 1 and 2). The curved portion lies between points A and B. The open curve corresponds to g = 0. The characteristic features of the distribution of $\bar{q}_w(s/H)$ for g = 0 are the presence of a maximum at points of reattachment and at the beginning of the bend (this maximum is related to the beginning of separation on the outer wall) and a minimum at the beginning of separation on the inner wall. The portion of the wall in the circulation region lies between points C and D.

For suction from the portion between A and B, $\bar{q}_w$ increases rapidly to a maximum $\bar{q}_w = \bar{q}_{w*}$ at the end of the suction region (curve 1 for g = −0.11). For injection (curve 2 for g = 0.09), on the other hand, $\bar{q}_w$ decreases rapidly.

This behavior of $\bar{q}_w$ is accounted for physically by the fact that for $g < 0$ the streamlines are compressed near the wall, and the temperature gradient along the normal to the wall increases. For $g > 0$ everything is opposite.

Figure 4 shows $\bar{q}_{w*}$ as a function of g for regimes I-IV. The values of $\bar{q}_{w*}$ for each regime correspond to a heat flux to the end of the suction or injection region (for suction $\bar{q}_{w*}$ is the maximum in the distribution of $\bar{q}_w$ along the wall, Fig. 3). The difference in curves I-IV is accounted for physically by the different initial (g = 0) distribution of T along the normal to the wall at different locations along the channel.

A comparison of Figs. 2 and 4 shows that the improvement of gasdynamic flow (decrease of pressure loss) during suction is accompanied by an undesirable increase of heat fluxes to the wall, and, during injection, on the other hand, a decrease in heat fluxes is accompanied by a worsening of the gasdynamics of the flow. Therefore, it is necessary to seek optimum regimes of flow control.

However, for $-0.05 < g < 0$, regime I is the best of regimes I-IV, since for identical flow rates of the sucked gas it leads to the largest decreases of losses and the lowest level of heat fluxes at the location of the suction.

NOTATION

x, y, longitudinal and transverse coordinates of Cartesian system in the rectilinear portions of the channel; s, length of inner wall, measured from the channel entrance; **n**, normal from wall into the flow; H, height of channel; r, θ, radial and angular variables of polar coordinate system in curved section; R, radius of curvature of inner wall; **V**, ρ, T, P, velocity, density, temperature, and static pressure of gas; u, v, longitudinal and transverse components of **V**; v_{nw}, normal component of gas velocity at the wall; s_1, s_2, coordinates of beginning and end of suction or injection portion; s_3, channel outlet; G_0, flow rate of gas in entrance section; G_i, flow rate of injected ($G_i > 0$) and sucked ($G_i < 0$) gas; μ, λ, c_{p0}, c_{v0}, dynamic viscosity, thermal conductivity, and specific heats; q_w, heat flux to wall; S, η, dimensionless geometric parameters characterizing the suction region; g, relative flow rate of sucked gas; Cu = H/R, curvature of inner wall; ξ, total pressure loss coefficient; $\bar{q}_w$, dimensionless heat flux to wall; $\bar{T}_w = T_w/T_0$, temperature factor; Re, Pr, and M, Reynolds, Prandtl, and Mach numbers calculated with values at inlet; V_w, relative normal velocity of injection and suction: Subscripts: w, wall; 0, channel inlet and axis.

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